ter plates translate the interaction region between the layers away from the two-dimensionalizing influence of the sharp shoulders, and as the vortex layers get thicker relative to the lateral interaction distance, and more contorted, the subsequent motion gets more and more incoherent in accordance with our expectation. The local oscillating pressures on the base are also undoubtedly relieved.

c) There is a confining influence because of the sidewalls of the tunnel, the span being only three times the base height. (Although the interaction in the side-wall boundary layers brings about some three-dimensional effects, the confining, two-dimensionalizing effect is thought to be more important at these Reynolds numbers.) The "softness" of the instability (Sec. 3) is reflected in the manner in which the changing boundary conditions influence the dimensionless frequency (multiplied by base height and divided by freestream speed). These so-called Strouhal numbers are 0.27, 0.30, and 0.24 in Figs. 1–3, respectively.

Roshko's phenomenon of the recovery of pronounced perodicity in the wake[7] of two-dimensional circular cylinders at "transcritical" Reynolds numbers could be more or less similar to state of affairs in Figs. 2 or 3, the splitter plates simulating the partial obstruction due to the circular base. In Roshko's case, an increase in coherence** is apparently due to the complete disappearance of several links in the instability chain, namely, of the laminar (or mixed laminar-turbulent) separation some 80°–90° from the front stagnation line, of the instability and/or transition of the separated layer, and of the consequent turbulent or mixed reattachment of the layer, which are present in the more random flows for an indefinite range of Reynolds number above approximately 200,000 on smooth circular cylinders. At Reynolds numbers past 3.5×10^6, the universal transition to turbulence upstream of the 80° station and the single turbulent separation in presence of a strong, fairly two-dimensional, adverse pressure gradient could be roughly comparable to the sequence of instabilities in Figs. 2 or 3. Again, we note that the turbulence of Roshko's boundary layers does not necessarily bring about complete randomness.

7) In conclusion, the writer feels that increased attention to the behavior of the whole vorticity envelope, and in particular to the subtle small influences (including that of the organizing motion of the body) on its multiple instabilities, should help to understand the less random aspects of these cases of "impure chaos."

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** Low freestream turbulence may have also helped. It is also possible that increased organization of wakes may occur only locally over spanwise segments of the cylinder and wake. Another possibility can be sketched in terms of the flow downstream of the right rim of Fig. 4: This nearly homogenized wake could undergo large-scale inviscid instability in its own right. Clearly, these many possibilities indicate the need for increased aerodynamic information in conjunction with structural load testing.

Strain-Displacement Relations in Large Displacement Theory of Shells

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Nomenclature

A_1, A_2	= functions of α_1, α_2
$B_1, B_2, \ldots B_6$	= functions of u, v, w
$C_1, C_2, \ldots C_6$	= functions of $\bar{u}, \bar{v}, \bar{w}$
$D_1, D_2, \ldots D_6$	= functions of θ, ψ, χ
H_1, H_2, H_3	= Lamé coefficients
R_1, R_2	= principal radii of curvature
X, Y, Z	= right-hand Cartesian coordinate system
u, v, w	= displacement components along the α_1, α_2, z directions, respectively, at an arbitrary point
$\bar{u}, \bar{v}, \bar{w}$	= displacement components along the α_1, α_2, z directions, respectively, on middle surface of shell
z	= coordinate in the direction of the normal to the shell surface
α_1, α_2	= curvilinear coordinate lines that are also lines of principal curvature of middle surface of shell
$\alpha_{31}, \alpha_{32}, \alpha_{33}$	= functions of $\bar{u}, \bar{v}, \bar{w}$
$\epsilon_{11}, \epsilon_{22}, \epsilon_{zz}, \epsilon_{12}, \epsilon_{1z}, \epsilon_{2z}$	= normal and shear strains at an arbitrary point
$\bar{\epsilon}_{11}, \bar{\epsilon}_{22}, \bar{\epsilon}_{12}$	= normal and shear strains on middle surface of shell
θ	= function of $\bar{u}, \bar{v}, \bar{w}$
$\nu_{11}, \nu_{22}, \nu_{12}$	= functions of $\bar{u}, \bar{v}, \bar{w}$
$\chi, \chi_{11}, \chi_{22}, \chi_{12}$	= functions of $\bar{u}, \bar{v}, \bar{w}$
ψ	= function of $\bar{u}, \bar{v}, \bar{w}$

I. Introduction

STRAIN-DISPLACEMENT relationships are one of the most important sets of formulas in structural analysis. An incorrect relationship will give rise to errors in the results and conclusions of various structural analyses, such as buckling, vibrations, or stress analysis. Using the large displacement theory and Kirchhoff's assumption on the preservation of the normal element, Ref. 1 derived a set of strain-displacement relationships. These relationships can be improved by 1) correction of an error made in the derivation and 2) clarification of the sign convention for the principal radii.

II. Analysis

Let the middle surface of the shell be defined by

$$X = X(\alpha_1, \alpha_2) \qquad Y = Y(\alpha_1, \alpha_2)$$
$$Z = Z(\alpha_1, \alpha_2)$$

where X, Y, Z are rectangular coordinates, and α_1, α_2 are

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curvilinear coordinate lines, which are also lines of principal curvature of the middle surface. And let z be the normal distance between the middle surface and any point in space. Also, let X, Y, Z and α_1, α_2, z be both right-hand coordinate systems. For the preceding curvilinear coordinate system, Ref. 2 shows that the Lamé coefficients are given by

$$H_1 = A_1\left(1 - \frac{z}{R_1}\right) \qquad H_2 = A_2\left(1 - \frac{z}{R_2}\right) \qquad H_3 = 1 \tag{1}$$

where

$$\left.\begin{aligned}
A_1 &= \left[\left(\frac{\partial X}{\partial \alpha_1}\right)^2 + \left(\frac{\partial Y}{\partial \alpha_1}\right)^2 + \left(\frac{\partial Z}{\partial \alpha_1}\right)^2\right]^{1/2}\\
A_2 &= \left[\left(\frac{\partial X}{\partial \alpha_2}\right)^2 + \left(\frac{\partial Y}{\partial \alpha_2}\right)^2 + \left(\frac{\partial Z}{\partial \alpha_2}\right)^2\right]^{1/2}\\
\frac{1}{R_1} &= \frac{1}{A_1{}^3A_2}\begin{vmatrix} \frac{\partial^2 X}{\partial \alpha_1{}^2} & \frac{\partial^2 Y}{\partial \alpha_1{}^2} & \frac{\partial^2 Z}{\partial \alpha_1{}^2}\\ \frac{\partial X}{\partial \alpha_1} & \frac{\partial Y}{\partial \alpha_1} & \frac{\partial Z}{\partial \alpha_1}\\ \frac{\partial X}{\partial \alpha_2} & \frac{\partial Y}{\partial \alpha_2} & \frac{\partial Y}{\partial \alpha_2}\end{vmatrix}\\
\frac{1}{R_2} &= \frac{1}{A_1A_2{}^3}\begin{vmatrix} \frac{\partial^2 X}{\partial \alpha_2{}^2} & \frac{\partial^2 Y}{\partial \alpha_2{}^2} & \frac{\partial^2 Z}{\partial \alpha_2{}^2}\\ \frac{\partial X}{\partial \alpha_1} & \frac{\partial Y}{\partial \alpha_1} & \frac{\partial Z}{\partial \alpha_1}\\ \frac{\partial X}{\partial \alpha_2} & \frac{\partial Y}{\partial \alpha_2} & \frac{\partial Z}{\partial \alpha_2}\end{vmatrix}
\end{aligned}\right\} \tag{2}$$

The coefficients A_1, A_2, R_1, R_2 satisfy the Gauss-Codazzi equations:

$$\left.\begin{aligned}
\frac{\partial}{\partial \alpha_1}\left(\frac{A_2}{R_2}\right) &= \frac{1}{R_1} \times \frac{\partial A_2}{\partial \alpha_1}\\
\frac{\partial}{\partial \alpha_2}\left(\frac{A_1}{R_1}\right) &= \frac{1}{R_2} \times \frac{\partial A_1}{\partial \alpha_2}\\
\frac{\partial}{\partial \alpha_1}\left(\frac{1}{A_1}\frac{\partial A_2}{\partial \alpha_1}\right) + \frac{\partial}{\partial \alpha_2}\left(\frac{1}{A_2}\frac{\partial A_1}{\partial \alpha_2}\right) &= -\frac{A_1A_2}{R_1R_2}
\end{aligned}\right\} \tag{3}$$

From the nonlinear theory of elasticity,[1] the following strain-displacement relations can be derived for small deformations (normal and shear strains) and arbitrary displacements (u, v, w) for the curvilinear coordinate system α_1, α_2, z. Here, the components of displacement along the α_1, α_2 and z directions, denoted by u, v, w, need not be small. Hence

$$\left.\begin{aligned}
\epsilon_{11} &= \frac{B_1}{1 - (z/R_1)} + \frac{1}{2[1 - (z/R_1)^2]}(B_1{}^2 + B_2{}^2 + B_3{}^2)\\
\epsilon_{22} &= \frac{B_4}{1 - (z/R_2)} + \frac{1}{2[1 - (z/R_2)^2]}(B_4{}^2 + B_5{}^2 + B_6{}^2)\\
\epsilon_{zz} &= \frac{\partial w}{\partial z} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right]\\
\epsilon_{12} &= \frac{B_2}{1 - (z/R_1)} + \frac{B_5}{1 - (z/R_2)} + \\
&\quad \frac{1}{[1 - (z/R_1)][1 - (z/R_2)]}(B_1B_5 + B_2B_4 + B_3B_6)\\
\epsilon_{2z} &= \frac{\partial v}{\partial z} + \frac{1}{1 - (z/R_2)} \times\\
&\quad \left[B_5\frac{\partial u}{\partial z} + B_4\frac{\partial v}{\partial z} + B_6\left(1 + \frac{\partial w}{\partial z}\right)\right]\\
\epsilon_{1z} &= \frac{\partial u}{\partial z} + \frac{1}{1 - (z/R_1)} \times\\
&\quad \left[B_1\frac{\partial u}{\partial z} + B_2\frac{\partial v}{\partial z} + B_3\left(1 + \frac{\partial w}{\partial z}\right)\right]
\end{aligned}\right\} \tag{4}$$

where

$$\left.\begin{aligned}
B_1 &= \frac{1}{A_1}\frac{\partial u}{\partial \alpha_1} + \frac{v}{A_1A_2}\frac{\partial A_1}{\partial \alpha_2} - \frac{w}{R_1}\\
B_2 &= \frac{1}{A_1}\left(\frac{\partial v}{\partial \alpha_1} - \frac{u}{A_2}\frac{\partial A_1}{\partial \alpha_2}\right)\\
B_3 &= \frac{1}{A_1}\frac{\partial w}{\partial \alpha_1} + \frac{u}{R_1}\\
B_4 &= \frac{1}{A_2}\frac{\partial v}{\partial \alpha_2} + \frac{u}{A_1A_2}\frac{\partial A_2}{\partial \alpha_1} - \frac{w}{R_2}\\
B_5 &= \frac{1}{A_2}\left(\frac{\partial u}{\partial \alpha_2} - \frac{v}{A_1}\frac{\partial A_2}{\partial \alpha_1}\right)\\
B_6 &= \frac{1}{A_2}\frac{\partial w}{\partial \alpha_2} + \frac{v}{R_2}
\end{aligned}\right\} \tag{5}$$

The expression for ϵ_{1z} given in Eqs. (VI. 26) of Ref. 1 contains an algebraic error.

Kirchhoff's assumption on the preservation of the normal element leads to the following:

$$\begin{aligned}
\epsilon_{1z} = \epsilon_{2z} = \epsilon_{zz} = 0 \qquad & u = \bar{u} + z\theta\\
v = \bar{v} + z\psi \qquad & w = \bar{w} + z\chi
\end{aligned} \tag{6}$$

where $\bar{u}$, $\bar{v}$, $\bar{w}$ are the displacements of the middle surface of the shell, and $\bar{u}$, $\bar{v}$, $\bar{w}$ are the displacements of the middle surface of the shell, and $\bar{u}$, $\bar{v}$, $\bar{w}$, θ, ψ, and χ are functions of α_1 and α_2 only.

Substituting Eqs. (6) into the third, fifth, and sixth of Eqs. (4), one obtains five equations of which only three are independent. These three equations are

$$\left.\begin{aligned}
\theta^2 + \psi^2 + (1 + \chi)^2 &= 1\\
(1 + C_1)\theta + C_2\psi + C_3(1 + \chi) &= 0\\
C_5\theta + (1 + C_4)\psi + C_6(1 + \chi) &= 0
\end{aligned}\right\} \tag{7}$$

where $C_1 \ldots C_6$ are obtained from $B_1 \ldots B_6$, respectively, by the transformation $u \to \bar{u}, v \to \bar{v}, w \to \bar{w}$, i.e., $C_3 = [(1/A_1) \times (\partial\bar{w}/\partial\alpha_1)] + \bar{u}/R_1$.

The two unknown variables θ and ψ can be eliminated from the three parts of Eq. (7) to give

$$(\alpha_{31}{}^2 + \alpha_{32}{}^2 + \alpha_{33}{}^2)(1 + \chi)^2 = \alpha_{33}{}^2 \tag{8}$$

where

$$\left.\begin{aligned}
\alpha_{31} &= C_2C_6 - C_3(1 + C_4)\\
\alpha_{32} &= C_3C_5 - C_6(1 + C_1)\\
\alpha_{33} &= (1 + C_1)(1 + C_4) - C_2C_5
\end{aligned}\right\} \tag{9}$$

Let

$$\chi = \chi_0 + \chi_{\text{I}} + \chi_{\text{II}} + \text{higher order terms} \tag{10}$$

where

χ_0 = const
χ_{I} = a linear function of $\bar{u}$, $\bar{v}$, $\bar{w}$ and their derivatives
χ_{II} = a second power function of $\bar{u}$, $\bar{v}$, $\bar{w}$ and their derivatives

Substituting Eq. (10) into (8) and equating terms of the same power, one obtains

$$\chi_0 = \chi_{\text{I}} = 0 \qquad \chi_{\text{II}} = -\tfrac{1}{2}(C_3{}^2 + C_6{}^2) \tag{11}$$

In Ref. 1, Eq. (8) is solved by the improper introduction of an approximation $\alpha_{31}{}^2 + \alpha_{32}{}^2 + \alpha_{33}{}^2 = 1$. The expression of χ thus obtained contains terms that are linear functions of the derivatives of $\bar{u}$ and $\bar{v}$. That expression of χ is erroneous and should be corrected by the use of the preceding expressions [Eqs. (10) and (11)].

Equations (7, 10, and 11) can be solved to give the following expressions of θ and ψ up to and including terms of second power:

$$\begin{aligned} \theta &= \alpha_{31} + C_3(C_1 + C_4) \\ \psi &= \alpha_{32} + C_6(C_1 + C_4) \end{aligned} \qquad (12)$$

Erroneous results of θ and ψ are given in Ref. 1.

Substituting Eqs. (6, 10, 11, and 12) into the first, second, and fourth of Eqs. (4), and making use of the approximation that the ratio z/R is small compared to unity, one has

$$\left.\begin{aligned} \epsilon_{11} &= \bar{\epsilon}_{11} + z\chi_{11} + z^2\nu_{11} \\ \epsilon_{22} &= \bar{\epsilon}_{22} + z\chi_{22} + z^2\nu_{22} \\ \epsilon_{12} &= \bar{\epsilon}_{12} + z\chi_{12} + z^2\nu_{12} \end{aligned}\right\} \qquad (13)$$

where

$$\left.\begin{aligned}
\bar{\epsilon}_{11} &= C_1 + \tfrac{1}{2}(C_1^2 + C_2^2 + C_3^2) \\
\chi_{11} &= [(C_1/R_1) + D_1] + (1/R_1)(C_1^2 + C_2^2 + C_3^2) + (C_1D_1 + C_2D_2 + C_3D_3) \\
\nu_{11} &= \frac{1}{R_1}\left[\left(\frac{C_1}{R_1}\right) + D_1\right] + \frac{3}{2R_1^2}(C_1^2 + C_2^2 + C_3^2) + \left(\frac{2}{R_1}\right)(C_1D_1 + C_2D_2 + C_3D_3) + \tfrac{1}{2}(D_1^2 + D_2^2 + D_3^2) \\
\bar{\epsilon}_{22} &= C_4 + \tfrac{1}{2}(C_4^2 + C_5^2 + C_6^2) \\
\chi_{22} &= [(C_4/R_2) + D_4] + (1/R_2)(C_4^2 + C_5^2 + C_6^2) + (C_4D_4 + C_5D_5 + C_6D_6) \\
\nu_{22} &= \left(\frac{1}{R_2}\right)\left(\frac{C_4}{R_2} + D_4\right) + \left(\frac{3}{2R_2^2}\right) \times (C_4^2 + C_5^2 + C_6^2) + \left(\frac{2}{R_2}\right) \times (C_4D_4 + C_5D_5 + C_6D_6) + \tfrac{1}{2}(D_4^2 + D_5^2 + D_6^2) \\
\bar{\epsilon}_{12} &= (C_2 + C_5) + (C_1C_5 + C_2C_4 + C_3C_6) \\
\chi_{12} &= \left(\frac{C_2}{R_1} + \frac{C_5}{R_2}\right) + (D_2 + D_5) + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)(C_1C_5 + C_2C_4 + C_3C_6) + (C_1D_5 + C_5D_1 + C_2D_4 + C_4D_2 + C_3D_6 + C_6D_3) \\
\nu_{12} &= \left(\frac{C_2}{R_1^2} + \frac{C_5}{R_2^2}\right) + \left(\frac{D_2}{R_1} + \frac{D_5}{R_2}\right) + \left(\frac{1}{R_1^2} + \frac{1}{R_1R_2} + \frac{1}{R_2^2}\right)(C_1C_5 + C_2C_4 + C_3C_6) + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)(C_1D_5 + C_5D_1 + C_2D_4 + C_4D_2 + C_3D_6 + C_6D_3) + (D_1D_5 + D_2D_4 + D_3D_6)
\end{aligned}\right\} \qquad (14)$$

The expressions $D_1 \ldots D_6$ are obtained from $B_1 \ldots B_6$, respectively, by the transformation $u \to \theta$, $v \to \psi$, $w \to \chi$, i.e., $D_3 = [(1/A_1)(\partial\chi/\partial\alpha_1)] + \theta/R_1$.

Because of the error in deriving the expressions for θ, ψ, χ, the strain-displacement relations given in Eqs. (VI. 41) and (VI. 42) of Ref. 1 are in error and should be revised. For the same reason, the corresponding formulas for the flat plate in Sec. 46 of Ref. 1 should be revised.

When both X, Y, Z and α_1, α_2, z are right-hand coordinate systems, Eqs. (2) automatically give the correct signs of the principal radii of curvature R_1 and R_2. The formulas given here for the general shell can be readily transformed to obtain the formulas for special shells, such as spherical, conical, cylindrical, or toroidal shells, for flat plates, and for curved and straight beams.

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Prandtl Number Dependence of Heat Transfer in Falkner-Skan Flow

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Some years ago, Spence[1] derived the Prandtl number (σ) dependence of the recovery and Reynolds-analogy factors for heat transfer to a flat plate with the density-viscosity product constant. He expanded the known temperature solution, first given by Pohlhausen, in a Taylor series in $(\sigma - 1)$, and expressed both factors as the product of 1) the value at $\sigma = 1$, 2) a power of σ, and 3) a series of the form $1 + K(\sigma - 1)^2 + \ldots$. The results confirmed the fitted $\sigma^{1/2}$ dependence of the recovery factor suggested by Pohlhausen and modified his suggested $\sigma^{-2/3}$ dependence of the Reynolds-analogy factor to $\sigma^{-0.64885}$. (It is interesting to note that $\sigma^{-0.65}$ had been suggested by Crocco[2] without any derivation.) The factor K, which indicates the accuracy of the power law, was also computed by Spence,[1] but given incorrectly in his paper.

The purpose of this note is to correct the values of K given by Spence, and to give similar results for the heat transfer to two-dimensional and axisymmetric stagnation points in incompressible flow.

Examination of Spence's equation (9) shows that the constant K given in Eq. (10) as $+ 0.0095$ should be -0.0095, so that the recovery factor r is

$$r = \sigma^{1/2}\,[1 - 0.0095\,(\sigma - 1)^2 + \ldots]$$

This is in better agreement with Pohlhausen's exact calculations, which show $\sigma^{1/2}$ to be higher than the exact values, not lower.

A more serious error occurs in Spence's Reynolds-analogy factor s, where Spence gives $K = -0.347$, which would indicate that the power law was not a very good approximation to s. Actually, this number contains two errors, one of which is incorrect addition below Eq. (13) where 0.69421 should be 1.01433. But this would lead to $K = -0.50716$, which is also wrong. The source of the second error is in the left side of the second equality in Eq. (13), where the factor 1 should be 2. When this correction is also applied, we find $K = -0.00716$, so that

$$s = \sigma^{-0.64885}\,[1 - 0.00716\,(\sigma - 1)^2 + \ldots]$$

Now the accuracy of the power law becomes clear.

Similar results can be obtained for the recovery factor and heat-transfer rate in other cases of Falkner-Skan flow, where the pressure gradient is not zero, though a somewhat different method must be used since the explicit solution of the energy equation is not known in general. However, since only expansions about $\sigma = 1$ are desired, only the solution for $\sigma = 1$ is needed, and this easily can be found numerically. Details of derivation in the general case are given by Kemp.[3]

The results for two particularly interesting cases will be given here, namely, the two-dimensional and axisymmetric stagnation points. Exact values for the former have been given by Squire,[4] for the latter by Sibulkin,[5] and both have suggested a $\sigma^{0.4}$ power law representation of the heat-transfer parameter. The corresponding expansions are (in the notation of Squire[4] and Sibulkin[5]), respectively,

$$\alpha_3 = 0.570\,\sigma^{0.3892}\,[1 - 0.0092\,(\sigma - 1)^2 + \ldots] \quad \text{(two-dimensional)}$$

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